

# Determining the Probability of a Good Day for Sowing Cotton

## Task 2 – Constructing and interpreting Venn diagrams and two-way tables

To plant cotton on a particular day, there are two important conditions:

Condition A: Soil temperature at 10 cm depth above 14°C at 9am (AEST)

Condition B: Forecast average temperatures for the week following planting on a rising plane.

A day in the cotton planting season may or may not meet these conditions.

When a condition is met, it is said to be **satisfied**.

In Task 1, a farmer decided to look at historical climate data to see how frequently these conditions have been satisfied at Trangie. He decided to look at data for the planting season in 1992 because it was the coolest planting season in the record. He also looked at data for the planting season in 2006 because it was the warmest planting season in the record.

In a planting season, the days when Condition A was satisfied form a **set** of days which will be called Set A. The days when Condition B was satisfied form another **set** of days which will be called Set B.

The Cotton Seed Distributors' website (<http://www.csd.net.au/greenlight>) says:

- If *both* conditions are satisfied, it is a **green light** day, meaning that the farmer should go ahead and sow cotton that day.
- If *neither* condition is satisfied, it is a **red light** day, meaning that the farmer should stop and not sow cotton that day.
- If *only one* of the two conditions is satisfied, it is an amber light day, meaning that the farmer should be cautious and only sow cotton that day if there is unlikely to be enough green light days remaining in the planting season.

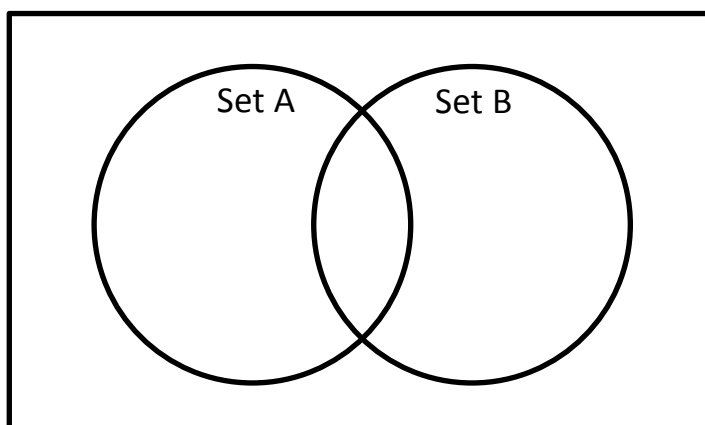
Days might be amber light days because *only* condition A is satisfied (**brown light** days) or because *only* condition B is satisfied (**orange light** days).

### Venn Diagrams

The information for each planting season can be shown in a Venn diagram. A Venn diagram shows the relationship between two or more sets, each set being represented by a circle.

The circle labelled Set A represents the number of days that satisfy Condition A.

The circle labelled Set B represents the number of days that satisfy Condition B.



# Determining the Probability of a Good Day for Sowing Cotton

The two circles in the Venn diagram intersect (i.e. overlap) because some days satisfy Condition A *and* Condition B and are therefore in both Set A *and* Set B.

- What colour light was given to these days?

On the Venn diagram above, shade the region (i.e. the area) where the circles overlap, with this colour of pencil.

On some days, *neither* Condition A *nor* Condition B was satisfied.

- What colour light was given to these days?

On the Venn diagram above, shade the region outside the circles (but inside the rectangle) with this colour.

On some days, Condition A *was* satisfied but Condition B *was not* satisfied.

- What colour light was given to these days?

On the Venn diagram above, shade the part of the circle labelled Set A that *does not* overlap with the circle labelled Set B.

On some days, Condition A *was not* satisfied but Condition B *was* satisfied.

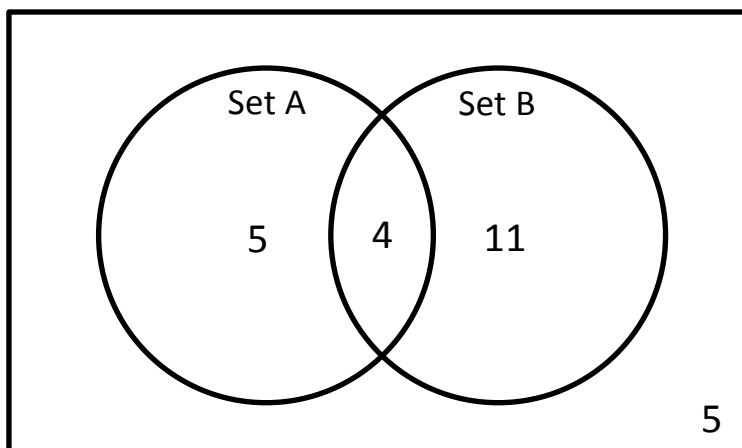
- What colour light was given to these days?

On the Venn diagram above, shade the part of the circle labelled Set B that *does not* overlap with the circle labelled Set A.

The table below shows the number of the different types of days found in the 1992 planting season.

	1992 Planting Season
Number of <b>green light</b> days	4
Number of <b>red light</b> days	5
Number of <b>brown light</b> days	5
Number of <b>orange light</b> days	11

Each number of days in the table matches a colour you used to shade a region of the previous Venn diagram. The Venn diagram below is the same diagram except a number (the number of days of each type) has been written in each region.



# Determining the Probability of a Good Day for Sowing Cotton

## Set Language and Notation

The items in a set are called the **elements** of the set.

The elements of the planting season at Trangie, are days.

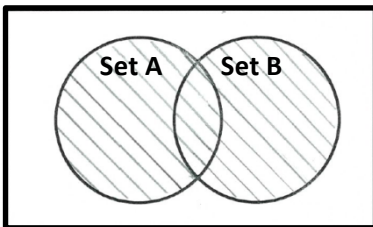
- What is the total number of days represented in the Venn diagram for the 1992 season?  
This number of elements is said to be the number in the **universal set**.

Look at the Venn diagram for the 1992 planting season.

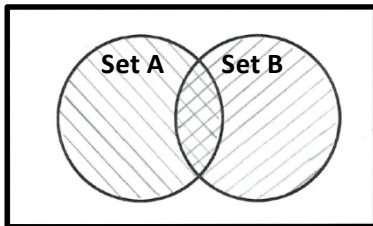
The **brown light** days and **green light** days are in Set A.

The notation **n(A)** means the number of elements in Set A.  $n(A) = 5 + 4 = 9$

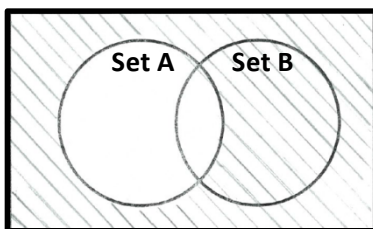
- What colour days are in Set B?
- What does  $n(B)$  equal?



The **union** of two sets, A and B (symbolised as  $A \cup B$ ) refers to all the elements from both sets. It is when *either* of the two conditions is satisfied. It can be found on a Venn diagram by shading both circles. It then includes every region of the diagram that's shaded.



The **intersection** of two sets, A and B (symbolised as  $A \cap B$ ) refers to the elements that are *common* to both sets. It is when *both* of the two conditions are satisfied. It can be found on a Venn diagram by shading one set in one direction and one set in the other direction (as shown). It is then the region shaded twice (hatched lines in both directions).

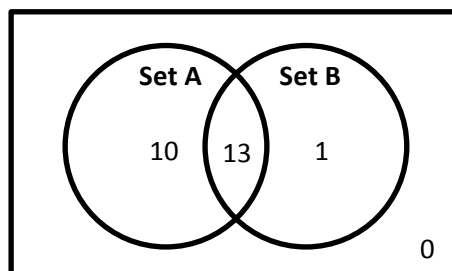


If an element is *not* within Set A, it is in the **complement** of Set A. The notation used for the complement of A, is  $A'$ . The complement of Set A, is Set  $A'$ . Set  $A'$  is the shaded region of this Venn diagram.

The Venn diagram below shows the data for the 2000 planting season.

- Shade it with a pencil to help you find the values of the following:

- $n(A \cup B) =$
- $n(A \cap B) =$
- $n(A' \cup B) =$
- $n(A' \cap B) =$
- $n(A' \cup B') =$



# Determining the Probability of a Good Day for Sowing Cotton

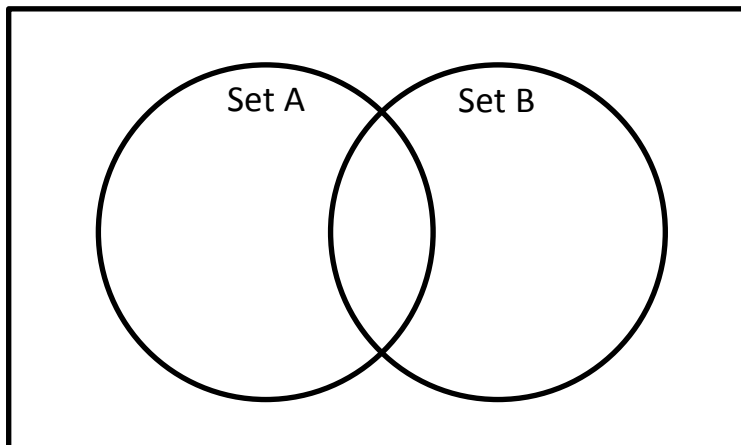
$$n(A' \cap B') =$$

- Is the rule  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$  true for the 2000 planting season?  
Is this rule true for the 1996 planting season?  
Is this rule true for *any* number of elements in Set A and Set B? Explain why or why not.  
Would this rule apply if there were *no green light* days (i.e. no overlap of the two sets)?

The table below shows the number of the different types of days found in the 2006 planting season at Trangie. This was the warmest planting season in the data record.

	2006 Planting Season
Number of <b>green light</b> days	12
Number of <b>red light</b> days	0
Number of <b>brown light</b> days	11
Number of <b>orange light</b> days	0

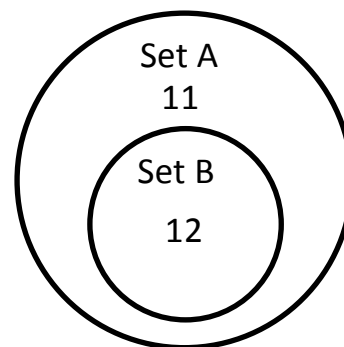
Represent this data by writing a number of days into each region of the Venn diagram below.



- How many days are in Set A?
- How many days are in Set B?

Because all the elements of Set B are in Set A, the diagram could be drawn as one circle inside the other.

- Colour each region according to the type of day it represents.
- Why do you think it is *not* necessary to place the circles inside a rectangle?



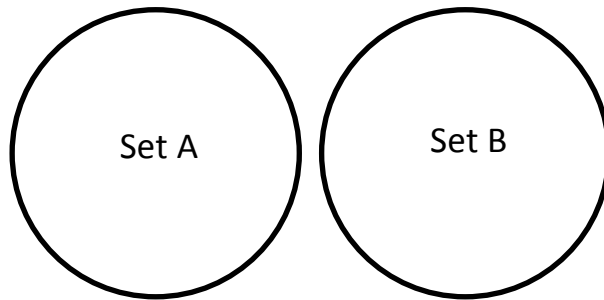
Sometimes sets have no elements that overlap.

# Determining the Probability of a Good Day for Sowing Cotton

These sets are described as being **mutually exclusive**.

This means that every element is either in Set A or in Set B, not both.

There is no intersection, as shown in the Venn diagram below.



	Set A	Set A'
Set B		
Set B'		

## Two-way Tables

Complement notation is often used as column and row headings in two-way tables.

The columns divide the days into the number of them that satisfy Condition A (Set A) and the number of them that *don't* satisfy Condition A (Set A').

The rows divide the days into the number of them that satisfy Condition B (Set B) and the number of them that *don't* satisfy Condition B (Set B').

Colour the 4 empty cells of the table above as follows.

**Green light** days satisfy Condition A and Condition B, so colour the cell that is in the Set A column *and* in the Set B row, green.

Colour the remaining 3 cells to indicate the sets in which there are **red light** days, **brown light** days and **orange light** days.

The table below shows the number of days in each cell.

1992	Set A	Set A'	Marginal Totals
Set B	4	11	
Set B'	5	5	
Marginal Totals			

In the table, write the total of each column and each row. These are called **marginal totals**.

# Determining the Probability of a Good Day for Sowing Cotton

- What does the total of Set B mean in this context?
- What does the total of Set A' mean in this context?
- What is the sum of the two column totals?
- What is the sum of the two row totals?
- Why should the sum of the columns totals, be the same as the sum of the row totals?

This sum is the number in the universal set.

- Using the Venn diagrams for 2000 and 2006, make a two-way table for each of these seasons.
  - Which of the three seasons (1992, 2000 or 2006) has the greatest proportion of days in Set B?
  - Did you expect all the seasons to have more than half of their days in Set B? Why or why not?